

Four-neutrino spectrum from oscillation data

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Abstract

It is shown that the Super-Kamiokande atmospheric up-down asymmetry, together with the results of all other neutrino oscillation experiments, allows to constraint the possible spectra of four massive neutrinos. The two schemes with two pairs of neutrinos with close masses separated by a gap of about 1 eV are favored by the data.

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The recent observation of an up-down asymmetry of high-energy μ -like events generated by atmospheric neutrinos in the Super-Kamiokande experiment [1] represents a convincing model-independent evidence in favor of neutrino oscillations. Indications in favor of disappearance of atmospheric ν_μ 's have been obtained also in the Kamiokande and IMB experiments and in the recent Soudan 2 and MACRO experiments. [2] Other indications in favor of neutrino oscillations have been obtained in solar neutrino experiments (Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande) and in the LSND experiment. [2] The flux of electron neutrinos measured in all five solar neutrino experiments is substantially smaller than the one predicted by the Standard Solar Model and a comparison of the data of different experiments indicate an energy dependence of the solar ν_e suppression, which represents a rather convincing evidence in favor of neutrino oscillations. [2] The accelerator LSND experiment is the only one that claims the observation of neutrino oscillations in appearance channels, specifically $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$.

The probabilities of neutrino oscillations depend on the elements of the neutrino mixing matrix U , that connects the flavor neutrino fields $\nu_{\alpha L}$ to the massive neutrino fields ν_{kL} through the relation $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}$, and on the phases $\Delta m_{kj}^2 L/E$, where $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$ (m_k is the mass of the neutrino field ν_k), L is the source-detector distance and E is the neutrino energy. If $\Delta m_{kj}^2 L/E \ll 1$ neutrino flavor transitions cannot be observed and if $\Delta m_{kj}^2 L/E \gg 1$ only the averaged transition probability can be measured. Since a variation of the transition probability as a function of neutrino energy has been observed in all the experiments mentioned above and the range of L/E probed by each type of experiment is different ($L/E \gtrsim 10^{10} \text{ eV}^{-2}$ for solar neutrino experiments, $L/E \sim 10^2 - 10^3 \text{ eV}^{-2}$ for atmospheric neutrino experiments and $L/E \sim 1 \text{ eV}^{-2}$ for the LSND experiment), it is clear that in order to explain all the observations with neutrino oscillations at least three Δm^2 's with different scales are needed: $\Delta m_{\text{sun}}^2 \lesssim 10^{-10} \text{ eV}^2$, $\Delta m_{\text{atm}}^2 \sim 10^{-3} - 10^{-2} \text{ eV}^2$, $\Delta m_{\text{LSND}}^2 \sim 1 \text{ eV}^2$ (if the MSW effect [2] is responsible of solar neutrino transitions, Δm_{sun}^2 must be smaller than about 10^{-4} eV^2 in order to have a resonance in the interior of the sun and is still at least one order of magnitude smaller than Δm_{atm}^2). This means that at least four light massive neutrinos must exist in nature. Here we consider the minimal possibility of four neutrinos, which implies that the three flavor neutrinos ν_e, ν_μ, ν_τ are accompanied by a sterile neutrino ν_s that does not take part in standard weak interactions.

The six types of four-neutrino mass spectra that can accommodate the hierarchy $\Delta m_{\text{sun}}^2 \ll \Delta m_{\text{atm}}^2 \ll \Delta m_{\text{LSND}}^2$ are shown in Fig. 1. In all these mass spectra there are two groups of close masses separated by the "LSND gap" of the order of 1 eV. In each scheme the smallest mass-squared difference corresponds to Δm_{sun}^2 (Δm_{21}^2 in schemes I and B, Δm_{32}^2 in schemes II and IV, Δm_{43}^2 in schemes III and A), the intermediate one to Δm_{atm}^2 (Δm_{31}^2 in schemes I and II, Δm_{42}^2 in schemes III and IV, Δm_{21}^2 in scheme A, Δm_{43}^2 in scheme B) and the largest mass squared difference $\Delta m_{41}^2 = \Delta m_{\text{LSND}}^2$ is relevant for the oscillations observed in the LSND experiment.

It has been shown [3] that the schemes I-IV are disfavored by the results of short-baseline accelerator and reactor disappearance neutrino oscillation experiments for all values of Δm_{41}^2 in the LSND-allowed range $0.2 - 2 \text{ eV}^2$, with the possible exception of the small interval from 0.2 to 0.3 eV^2 where there are no data from ν_μ short-baseline disappearance experiments. Here we will show [4] that this gap is closed by the inclusion in the analysis of the asymmetry of μ -like high-energy events

$$\mathcal{A} = (U - D)/(U + D) = -0.311 \pm 0.043 \pm 0.01 \quad (1)$$

measured in the Super-Kamiokande experiment. [1] Here U and D are the number of events in the zenith angle intervals $-1 < \cos \theta < -0.2$ and $0.2 < \cos \theta < 1$, respectively.

In the following we will consider only the scheme I with a mass hierarchy, but the results apply also to the schemes II, III and IV. Let us remark that in principle one could check which scheme is allowed by doing a combined fit of all data. However, at the moment it is not possible to perform such a fit because of the enormous complications due to the presence of many parameters (six mixing angles, etc.) and to the difficulties involved in a combined fit of the data of different experiments, which are usually analyzed by the experimental collaborations using different methods. Hence, we think that it is quite remarkable that one can exclude the schemes I–IV with the following relatively simple procedure.

The exclusion plots obtained in short-baseline $\bar{\nu}_e$ and ν_μ disappearance experiments imply that [3] $|U_{\alpha 4}|^2 \leq a_\alpha^0$ or $|U_{\alpha 4}|^2 \geq 1 - a_\alpha^0$ for $\alpha = e, \mu$, with [2] $a_e^0 \lesssim 4 \times 10^{-2}$ for $\Delta m_{41}^2 \gtrsim 0.1 \text{ eV}^2$ and $a_\mu^0 \lesssim 0.2$ for $\Delta m_{41}^2 \gtrsim 0.4 \text{ eV}^2$. However, since the survival probability of solar ν_e 's is bounded by [3] $P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \geq |U_{e4}|^4$, only the range

$$|U_{e4}|^2 \leq a_e^0 \quad (2)$$

is acceptable. In a similar way, since the survival probability of atmospheric ν_μ 's and $\bar{\nu}_\mu$'s (all the following inequalities are valid both for neutrinos and antineutrinos) is bounded by [3] $P_{\nu_\mu \rightarrow \nu_\mu}^{\text{atm}} \geq |U_{\mu 4}|^4$, it is clear that large values of $|U_{\mu 4}|^2$ are incompatible with the observed asymmetry (1).

Let us derive the upper bound for $|U_{\mu 4}|^2$ that follows from the asymmetry (1). Because of the small value of $\Delta m_{\text{atm}}^2 = \Delta m_{31}^2$, downward-going neutrinos do not oscillate with the atmospheric mass-squared difference and the survival probability of downward-going neutrinos given by

$$P_{\nu_\alpha \rightarrow \nu_\alpha}^D = |U_{\alpha 4}|^4 + (1 - |U_{\alpha 4}|^2)^2. \quad (3)$$

The conservation of probability and Eq.(2) allow to deduce the upper bound

$$P_{\nu_e \rightarrow \nu_\mu}^D \leq 1 - P_{\nu_e \rightarrow \nu_e}^D = 2|U_{e4}|^2(1 - |U_{e4}|^2) \leq 2a_e^0(1 - a_e^0). \quad (4)$$

From Eqs.(3) and (4) we obtain the upper bound

$$D \leq N_\mu \left[|U_{\mu 4}|^4 + (1 - |U_{\mu 4}|^2)^2 \right] + 2N_e a_e^0(1 - a_e^0), \quad (5)$$

where N_μ and N_e are number of muon (electron) neutrinos and antineutrinos produced in the atmosphere. On the other hand, taking into account only the part of D which is determined by the survival probability of ν_μ 's, we obtain the lower bound

$$D \geq N_\mu \left[|U_{\mu 4}|^4 + (1 - |U_{\mu 4}|^2)^2 \right]. \quad (6)$$

Furthermore, using the lower bound [3] $P_{\nu_\mu \rightarrow \nu_\mu}^{\text{atm}} \geq |U_{\mu 4}|^4$, for upward-going neutrinos we have

$$U \geq N_\mu |U_{\mu 4}|^4. \quad (7)$$

With the inequalities (5), (6) and (7), for the asymmetry (1) we obtain

$$-\mathcal{A} \leq \frac{(1 - |U_{\mu 4}|^2)^2 + 2 a_e^0(1 - a_e^0)/r}{(1 - |U_{\mu 4}|^2)^2 + 2 |U_{\mu 4}|^4}, \quad (8)$$

where $r \equiv N_\mu/N_e \simeq 2.8$. Solving the inequality (8) for $|U_{\mu 4}|^2$, we finally obtain the upper bound

$$|U_{\mu 4}|^2 \leq \frac{1 + \mathcal{A} - \sqrt{-2[\mathcal{A}(1 + \mathcal{A}) + (1 + 3\mathcal{A})a_e^0(1 - a_e^0)/r]}}{1 + 3\mathcal{A}} \equiv a_\mu^{\text{SK}}. \quad (9)$$

Since the measured value (1) of \mathcal{A} implies that $-\mathcal{A} \geq 0.254$ at 90% CL, from the inequality (8) we obtain the upper bound depicted by the horizontal line in Fig. 2 (the vertically hatched area is excluded).

In Fig. 2 we have also shown the bound $|U_{\mu 4}|^2 \leq a_\mu^0$ or $|U_{\mu 4}|^2 \geq 1 - a_\mu^0$ obtained from the exclusion plot of the short-baseline CDHS ν_μ disappearance experiment, which exclude the shadowed region.

The results of the LSND experiment imply a lower bound $A_{\mu;e}^{\text{min}}$ for the amplitude $A_{\mu;e} = 4|U_{e4}|^2|U_{\mu 4}|^2$ of $\nu_\mu \rightarrow \nu_e$ oscillations, from which we obtain the constraint

$$|U_{\mu 4}|^2 \geq A_{\mu;e}^{\text{min}}/4a_e^0. \quad (10)$$

This bound is represented by the curve in Fig. 2 labelled LSND + Bugey (the diagonally hatched area is excluded).

From Fig. 2 one can see that, in the framework of scheme I, there is no range of $|U_{\mu 4}|^2$ that is compatible with all the experimental data. Hence, the scheme with four neutrinos and a mass hierarchy is strongly disfavored.

The incompatibility of the experimental results with the mass spectrum I is shown also in Fig. 3, where we have plotted in the $A_{\mu;e}-\Delta m_{41}^2$ plane the upper bound $A_{\mu;e} \leq 4a_e^0 a_\mu^0$ for $\Delta m_{41}^2 > 0.26 \text{ eV}^2$ and $A_{\mu;e} \leq 4a_e^0 a_\mu^{\text{SK}}$ for $\Delta m_{41}^2 < 0.26 \text{ eV}^2$ (solid line, the region on the right is excluded). One can see that this constraint is incompatible with the LSND-allowed region (shadowed area).

The procedure presented above for the scheme I applies also to the schemes II, III and IV, in which there is a group of three close neutrino masses separated from the fourth mass by the LSND gap. Hence, we conclude that these schemes are disfavored. Only the four-neutrino schemes A and B in Fig. 1 are compatible with the results of all neutrino oscillation experiments.

REFERENCES

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FIGURES

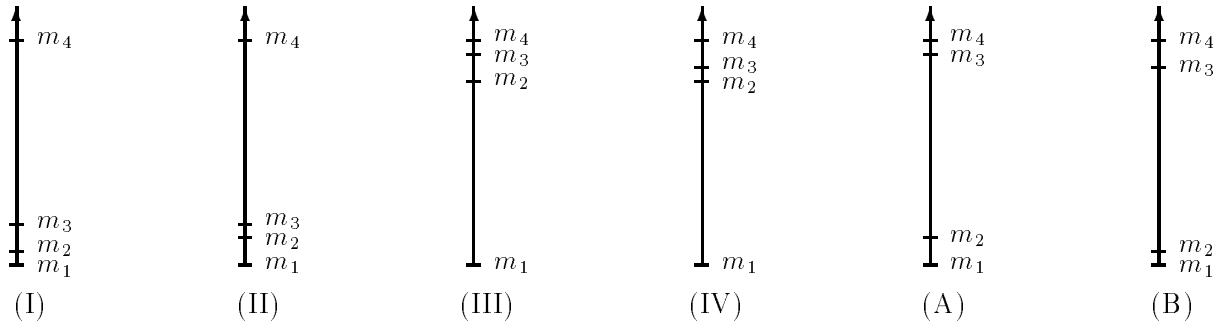


FIG. 1.

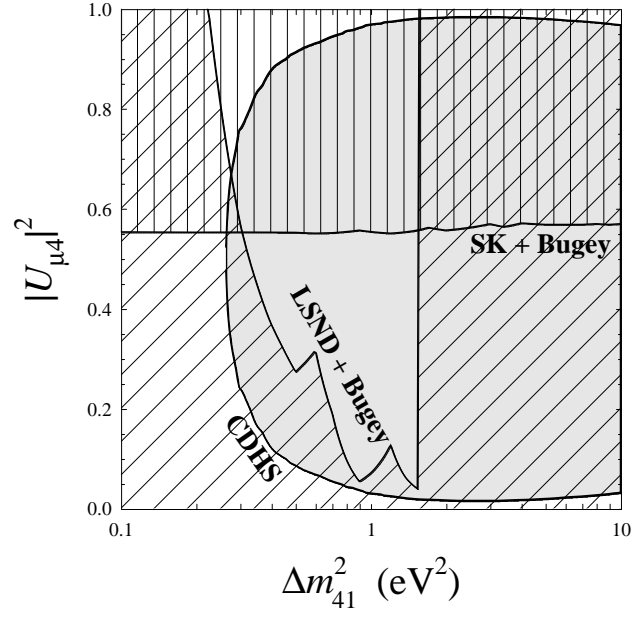


FIG. 2.

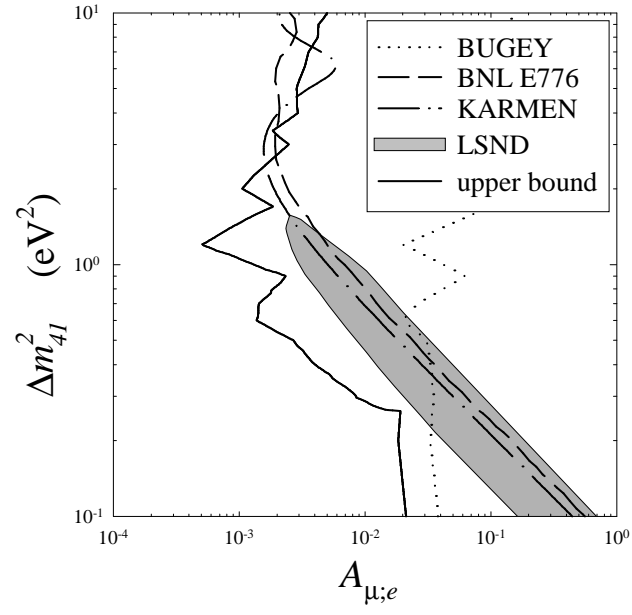


FIG. 3.